USING SPATIAL SYMMETRIES TO DO RESOURCE ALLOCATION IN LARGE SCALE SYSTEMS
OBJECTIVES

SOLVE THE RESOURCE ALLOCATION PROBLEM FOR LARGE SCALE LINEAR SYSTEMS. (CONTROL WITH FLOW CONSTRAINTS)

USE SYSTEM SPATIAL SYMMETRIES TO “BREAK” CONTROLLER DESIGN AND IMPLEMENTATION INTO SMALLER PROBLEMS

USE SYSTEM SPATIAL SYMMETRIES AGAIN TO ISOLATE FLOW CONSTRAINTS
MOTIVATION

LARGE SCALE SYSTEM

SPATIAL SYMMETRIES

DECOUPLING INTO SMALLER SYSTEMS

SMALL SCALE SYSTEM

SMALL SCALE SYSTEM

SMALL SCALE SYSTEM

SMALL SCALE SYSTEM
EXAMPLES OF SYSTEMS WITH SYMMETRY

DYNAMICS IS INVARIANT UNDER 90° ROTATION
EXAMPLES OF SYSTEMS WITH SYMMETRY

INVARIANT UNDER REFLECTION AND PERMUTATIONS
EXAMPLES OF SYSTEMS WITH SYMMETRY

EACH SMALL SCALE SYSTEM GOVERNS THE DYNAMICS OF THE LARGE SCALE SYSTEM OVER A SPATIAL EIGENVECTOR.

THE COMBINED EFFECT OF ALL SMALL SCALE SYSTEMS DESCRIBES THE DYNAMICS OF THE OVERALL SYSTEM.

EXAMPLES OF SPATIAL EIGENVECTORS ARE:

\[
\begin{pmatrix}
1 & 1 & 1 & \ldots & 1
\end{pmatrix} \\
\begin{pmatrix}
1 & -1 & 0 & \ldots & 0 & 0
\end{pmatrix}
\]

PHYSICAL MEANING:
SPATIAL MEAN VALUE OVER ALL SECTORS.

PHYSICAL MEANING:
SHIFT OF RESOURCES FROM SECTOR 1 TO 2.
ISOLATING FLOW CONSTRAINTS

SMALL SCALE SYSTEM  \(\leftrightarrow\)  SPATIAL EIGENVECTOR  \(\leftrightarrow\)  FREE

SMALL SCALE SYSTEM  \(\leftrightarrow\)  SPATIAL EIGENVECTOR  \(\leftrightarrow\)  FLOW CONSTR.

\(\cdots\)  \(\cdots\)  \(\cdots\)  \(\cdots\)  \(\cdots\)  \(\cdots\)  \(\cdots\)

SMALL SCALE SYSTEM  \(\leftrightarrow\)  SPATIAL EIGENVECTOR  \(\leftrightarrow\)  FREE

ALL FREE
ISOLATING FLOW CONSTRAINTS

LINEAR FLOW CONSTRAINTS CAN BE EASILY DEALT WITH, WHENEVER THEY CAN BE ISOLATED IN ONE OF THE SPATIAL EIGENVECTORS

AS AN EXAMPLE, THE FOLLOWING FLOW CONSTRAINT:

$$\sum_{i} \text{flow}_{\text{sector } i} = \text{constant}$$

COMpletely constrains the behaviour of the system along the following spatial eigenvector:

$$[1 \ 1 \ 1 \ \ldots \ 1 \ 1 \ 1]$$

The remaining spatial eigenvectors are not influenced by the constraint, provided that they orthogonal to $[1 \ldots 1]$. 
ISOLATING FLOW CONSTRAINTS

BOTH EXAMPLES ADMIT A SPATIAL EIGENVECTOR ASSOCIATED WITH THE TOTAL FLOW CONSTRAINT:

\[ \sum_{i} \text{flow}_{\text{sector } i} = \text{constant} \]
CONTROLLER DESIGN PROCEDURE

1. **Compute Spatial Eigenvectors**
2. **Choose Physically Meaningful Decoupling Eigenvectors**
   - One of the eigenvectors must be associated with the flow const.
3. **Design Independently One Controller for Each Free Spatial Eigenvector (Not Associated with Flow Constr.)**
CONTROLLER IMPLEMENTATION

ASSUME N SMALL SCALE SYSTEMS AND ONE FLOW CONSTRAINT. THIS LEADS TO N-1 INDEPENDENT CONTROLLERS.
SIMULATION EXAMPLE

\[ \sum_{i} flow_{sector\ i} = constant \]

4 SPATIAL EIGENVECTORS:

3 ACCOUNT FOR THE RESOURCE TRANSFER BETWEEN NEIGHBORING SECTORS

1 ACCOUNT FOR THE FLOW CONSTRAINT:
SIMULATION EXAMPLE: SECTOR MODEL

\[
\begin{align*}
\text{FRIENDLY} & \\
\dot{x}_1^f &= -a_1^f x_1^f + a_2^f x_2^f - a_e^f x_1^e \\
\dot{x}_2^f &= -b_2^f x_1^f - b_e^f x_e^f + u \\
\text{ENEMY} & \\
\dot{x}_1^e &= -a_1^e x_1^e + a_2^e x_2^e - a_f^e x_1^f \\
\dot{x}_2^e &= -b_2^e x_1^e - b_f^e x_e^f + d
\end{align*}
\]
ENGAGING THE ENEMY

SIMULATION EXAMPLE: SETUP

USING 3 H-INF CONTROLLERS TO MINIMIZE THE “AMPLIFICATION” FROM THE DISTURBANCE \( d \) TO A LINEAR FUNCTION OF THE MAIN RESOURCES.

3 DIFFERENT OBJECTIVES WERE SIMULATED:

- ENGAGING THE ENEMY
- NEUTRALIZE ENEMY PLAN
- REDUCE INFLUENCE OF ENEMY STRATEGIES
SIMULATION EXAMPLE: DISTURBANCE

IN THE FOLLOWING SIMULATION SETUP, THE ENEMY’S ALLOCATION IS REGARDED AS AN UNKNOWN DISTURBANCE.

THE SUBSEQUENT RESULTS WERE OBTAINED ASSUMING THE ENEMY DECIDED TO TRANSFER RESOURCES FROM SECTOR 1 TO 4.
SIMULATION EXAMPLE: RESULTS - ENGAGING THE ENEMY

SECTOR 1

SECTOR 2

SECTOR 3

SECTOR 4

(Time)
SIMULATION EXAMPLE: RESULTS - ENGAGING THE ENEMY

SECTOR 1

SECTOR 2

sector disturbance. control

SECTOR 3

SECTOR 4

(Time)
SIMULATION EXAMPLE: RESULTS - ENGAGING THE ENEMY

COMMENTS:

**BY INSPECTION IT IS POSSIBLE TO INFER:**

- (SLIDE 16) THAT THE FRIENDLY’S MAIN RESOURCE TRACKS CLOSELY THE ENEMY’S MAIN RESOURCE.

- (SLIDE 17) THAT THE FRIENDLY’S ALLOCATION IS VERY SIMILAR TO THE ENEMY’S ALLOCATION. THIS SHOWS THAT FROM THE MEASUREMENTS OF MAIN RESOURCES, THE CONTROLLER IS ABLE TO FIGURE OUT WHAT WAS THE ENEMY’S STRATEGY AND ACT ACCORDINGLY.
SIMULATION EXAMPLE: RESULTS - ENGAGING THE ENEMY

DECOUPLED CONTROLLERS

CONTROLLER 1
FLUX FROM SECTOR 1 TO 2

CONTROLLER 2
FLUX FROM SECTOR 2 TO 3

CONTROLLER 3
FLUX FROM SECTOR 3 TO 4

(Time)
SIMULATION EXAMPLE: RESULTS - ENEMY REDUCTION

SECTOR 1

SECTOR 2

SECTOR 3

SECTOR 4

(Time)
SIMULATION EXAMPLE: RESULTS - FRIEND. REDUCTION

SECTOR 1

SECTOR 2

SECTOR 3

SECTOR 4

(ENEMY’S RES.
FRIEND. RES.)

(Time)